Dynamic modelling of agricultural policies: the role of expectation schemes

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**Introduction**

Computable general equilibrium (CGE) models are extensively used to evaluate the economic effects of agricultural policies. A large part of these models, among which the Global Trade Analysis Project (GTAP) described in Hertel (1997) is one of the most popular, are static. Some CGE models are however said to be dynamic but most of them are in fact built as a succession of static models just linked by a jumping variable (mainly capital accumulation). This is the case of the Linkage model from the World Bank (van der Mensbrugghe, 2006) or the Mirage model from the CEPII (Bchir et al., 2002). These models don’t take into account the inter temporal decision process of economic agents. One of the important features of the agricultural sector is yet that there is a time lag between production decisions and harvests. This time lag implies that producers have to base their decision on expected rather than on observed market prices, and their possible expectation errors can induce price fluctuations. This phenomenon, formalised by Ezekiel (1938) in his famous Cobweb theorem, has often been used to justify the public intervention on agricultural markets (Mazier, 2003).

By ignoring the inter temporal dimension of agricultural producers’ decisions, the aforementioned models are not able account for the formation of their expectations. Indeed in these models economic agents are implicitly supposed to make rational expectations in the Muth sense, which means that their predictions are assumed to be “the same as the predictions of the relevant economic theory” (Muth, 1961). Of course this assumption is unsustainable, notably in the light of the recent food crisis. Furthermore it finds little support in the econometric literature which mainly concludes that agricultural producers make quasi rational expectations on output prices (Chavas, 1999). It thus seems interesting to study the economic effects of agricultural policies under expectation schemes more realistic than the rational one.

The case of non rational expectations has been quite largely debated in the economic literature, especially during the 70’s (Turnovsky, 1974, Mahé 1977, Hazell et Scandizzo,
One of the points that come out of this literature is that non rational expectations schemes can lead to chaotic variations of prices (Boussard, 1996). Thus, if rational expectations appear to be unrealistic, so can be non rational ones because of the huge fluctuations of prices and quantities they induce. Elsewhere an econometric analysis concerning the past distribution of agricultural prices might provide some piece of information about the way expectations are formed, but the interference of agricultural policies conducted until now really complicates the issue.

Our objective in this paper is to study the impacts of different expectation schemes on the economic evaluation of agricultural policies. For that purpose we want to apply a framework widely used to assess the effects of agricultural policies and to take into account the intertemporal dynamic decisions of producers. That is why we depart from the static GTAP model and convert it to a dynamic model under different assumption about the way farmers as well as other economic agents form their expectations. The effects of a shock concerning the agricultural policy are then examined under the different assumptions. What we find is that non rational expectations can actually lead to chaotic variations of prices, but this is mainly due to high supply elasticities which are less relevant in the short term than in the long term. Besides we show that the results from a dynamic model with rational expectations don’t differ much from those of a static model.

The paper is organized as follows. In a first part we describe the standard static GTAP AGR model and highlight its shortcomings concerning the need today to model the dynamic decision process of producers. In a second part we propose a new dynamic version of this model where we consider that all expectations are rational and compare its results to those of the static version. In a third part we move to the case of non rational expectations and again compare the results of this model to the previous ones. Finally we conclude.
1. Description and limitations of the standard GTAP AGR model

The following notations will be used in this section and in the rest of the paper:

**Index**
- \( i \): product
- \( r \): region

**Variables**
- \( Y_{ir} \): production
- \( L_{ir} \): labour
- \( K_{ir} \): capital
- \( w_{ir} \): labour income
- \( w_{kr} \): capital income
- \( Q_{ir} \): consumption
- \( E_r \): income
- \( P_{ir} \): price
- \( I_{ir} \): investment
- \( P_{I_{ir}} \): investment cost
- \( M_{ir} \): net imports (imports – exports)
- \( B_r \): balance of trade (net exports)
- \( S_r \): savings
- \( D_r \): foreign debt

1.1 Description of the model

The GTAP model is a CGE model initially developed at Purdue University and continuously improved by contributions from all over the world. It is a powerful model often used to value the effects of economic policies. Indeed it allows explaining trade flows of various products across several regions of the world. Today there exist different versions of the original GTAP model described in Hertel (1997). Reduces to a skeleton, this model is made of the following equations:

\[
P_{ir} Y_{ir} = w_{ir} L_{ir} + w_{kr} K_{ir} \quad \bot Y_{ir}
\]

\[
L_{ir} = L_{ir} \left( Y_{ir}, w_{ir}, w_{kr} \right) \quad \bot L_{ir}
\]

\[
K_{ir} = K_{ir} \left( Y_{ir}, w_{ir}, w_{kr} \right) \quad \bot K_{ir}
\]

\[
w_{ir} = w_{ir} \quad \bot w_{ir}
\]

\[
w_{kr} = w_{kr} \quad \bot w_{kr}
\]

\[
\bar{L}_{r} = \sum_i L_{ir} \quad \bot w_{l_{r}}
\]

\[
\bar{K}_{r} = \sum_i K_{ir} \quad \bot w_{k_{r}}
\]

\[
Q_{ir} = Q_{ir} \left( E_{r}, P_{r} \right) \quad \bot Q_{ir}
\]
Output quantities are determined by the zero profit condition (eq 1) and the quantities of factor used by the production costs minimisation (eq 2 and 3). In order to clarify the presentation, production factors here are assumed to be perfectly mobile, which leads to an equality of their remuneration (eq 4 and 5). Households own endowment commodities (labour and capital), so the regional income corresponds to the factors income (eq 9) which can be determined thanks to the fixity of factors (eq 6 and 7). Prices are derived from the market equilibrium condition (eq 10).

We focus here on a version of the GTAP model which incorporates a detailed representation of the agricultural sector in terms of sectoral coverage and elasticities specification: the GTAP AGR model. So, the main differences between the GTAP and GTAP AGR models essentially lie in the representation of the land factor, in the lower mobility of agricultural primary factors and in the values of some substitution elasticities between inputs.

As the standard GTAP model, the GTAP AGR model is static: it offers a picture of the international economic situation at a given time and allows simulating the long term effects of exogenous shocks like agricultural policies reforms. However, the risk, that is the fluctuation, of agricultural markets has become a more and more important point with the evolution of agricultural policies and this is particularly true with the Common Agricultural Policy (CAP) of the EU or the Farm Bill Bill of the US. For instance, since its creation the CAP has evolved from a price intervention scheme toward a system of payments more and more decoupled from production and prices. This gradual suppression of price supports will have several kinds of impacts. One of them is that European agricultural markets will probably be subject to higher fluctuations (Stanley et al., 2000, Stanley et al., 2002) which could modify the behaviour of farmers. Another consequence is that the suppression of price supports will make it more difficult for farmers to anticipate the prices at which they will sell their products. These
two features cannot be taken into account with a static model like GTAP AGR, notably because it is static.

1.2 Representation of investment and saving behaviours in the static model

One element we have not advocated yet concerning the GTAP AGR model and which becomes a crucial point when dealing with dynamic issues in CGE modelling is the representation of investment and saving behaviours. Indeed, these behaviours allow creating a link between periods in all existing dynamic CGE models. Before seeing how this link between periods occurs exactly, we focus in this subsection on the way investment and savings are modelled in static CGE models.

As pointed out by Dewatripoint and Michel (1986) the specification of investment is one of the key elements in the closure problem of CGE models: a neoclassical solution to this problem is to fix investment, another one is to allow investment to adjust to changes in savings.

In the standard GTAP model, as well as in the GTAP AGR version, a global bank is designed to mediate between global savings and regional investment (Hertel, 1997). This global bank collects all regional investment goods and sells them to regional households in order to satisfy their demand for savings. The equality between global investment and savings is then ensured through the real exchange rates which are endogenous. There exists an alternative specification of investment implemented in the GTAPinGAMS program (Rutherford, 1997). In this specification investment in a region is financed by savings, trade surplus and interest received on foreign debt in that region: \( PI_r I_r - S_r = r_r D_r + B_r \). Yet, as the model is static, the foreign debt is constant so the trade deficit equalizes the interests received on the foreign debt: \( r_r D_r = -B_r \) and in GTAP AGR \( B_r \) is fixed so \( r_r D_r \) is fixed too and we get: \( PI_r I_r = S_r \). The value of investment is thus implicitly equal to the value of savings and is fixed.

As we will use the GTAPinGAMS model for the applications in this paper we rely on this “fixed investment” closure rule. Nevertheless as shown by Killkenny and Robinson (1990) the choice of one closure rule or another has in fact few impacts for the analysis of agricultural policies.
1.3 Limitations of existing dynamic CGE models

As already pointed out, one of the limitations of the GTAP AGR model to study the effects of some agricultural policies is that it is static. There exists a dynamic version of the GTAP model: GTAP Dyn which was developed by Ianovichina and McDougall (Ianovichina and McDougall, 2000). This model focuses on the modelling of capital markets and the dynamics occur through a capital accumulation from one period to another: new investment at one period will increase the capital stocks for the next period. Using capital accumulation as a link between periods is quite usual way to introduce dynamics in CGE modelling. In fact to our knowledge it is the case for all the existing dynamic CGE models (van der Mensbrugghe, 2005, Devarajan and Go, 1998, Diao and Somwaru, 2000, Francois et al., 1996).

Contrary to the static model, in the GTAP Dyn model investment is determined endogenously as a function of the expectations of investors concerning the growth rate of capital rate of returns and these expectations exogenous: one expectation is associated to each region at each period. However, this setting imposes that economic agents base their decision on the characteristics concerning the current period and not all the periods: there is no inter temporal decision making. This is also the case of the Linkage model (van der Mensbrugghe, 2005) for instance, and of a large part of the so called dynamic CGE models. However there exist some models including this inter temporal process, these are notably the models of Devarajan and Go (1998) and Diao and Somwaru (2000). In these models expectations of all agents are supposed to be rational.

One objective in this paper is to create a dynamic CGE model incorporating inter temporal behaviours of economic agents and different expectation schemes from agricultural producers. To do this we will in a first step rely on the framework used by Devarajan and Go (1998) or Diao and Somwaru and create a dynamic model assuming that all economic agents behave rationally, this is the purpose of the second part of the paper.
2. Dynamic CGE model with rational expectations

A temporal dimension indexed $t$ is now added to each variable.

2.1 Capital and foreign debt accumulation

One of the first changes brought to the static GTAP AGR model is to endogenise the “dynamic” variables: investment and savings, taking into account the evolution of capital stocks and foreign debt. These last variables are thus not constant anymore but evolve from one period to another.

So, the foreign debt increase (decrease) with the interest received on the debt and the trade surplus (deficit) of the previous period: $D_{t+1} - D_t = rD_t + B_{t+1}$. Thus the interest don’t equalize the trade deficit anymore and equation (8) which determines the regional income in the static model becomes:

$$ E_{t+1} = w_l L_{t+1} + w_k K_{t+1} - rD_t $$

Then, as we already mentioned and like what is done in all the dynamic CGE models we know, we introduce a capital accumulation in each sector: at a given time in each region a part of the revenue is saved and is used to finance investment in each sector of the region. This investment increases the capital stock of the sector for the next period:

$$ K_{t+1} = K_t + (1 - \delta_t) K_t - I_t $$

with $\delta_t$ the capital depreciation rate. So, equation (3) which determines the capital in each sector in the static model now determines the capital income and equation (5) is suppressed.

The capital accumulation equation implies that the capital increases as time goes by if the investment is higher than the capital depreciation. Yet, as we will see below, the investment is increasing in the capital rate of return and the capital rate of return is decreasing in the capital stock. That is why the capital stock increase stops at a given time (Francois et al., 1996) and from this time the capital stock remains constant. This is called the steady state. We denote $T$ the first period of this steady state which is thus characterized (Diao and Somwaru, 1997) by the fact that the investment exactly compensate for the capital depreciation: $I_{t=T} = \delta_t K_{t=T}$, the cost (and consequently the value) of investment is
stabilized: $P_{it}^{T} = P_{it}^{T-1}$, and finally the foreign debt remains constant too which means that the interests on the debt equalize the trade deficit. These characteristics are in fact those of the static model which means that the standard GTAP AGR model indeed offers a representation of the world markets at a steady state. The dynamic model we built here allows the evolution of these markets from one steady state to another one after a shock.

2.2 Inter temporal decisions

Producer’s decisions

To decide how much to invest the producer has to consider that the investment made during one period will increase the capital stock of his firm for the next periods. This increase in capital stock will allow him to produce more and increase his profit. So, to take his investment decision the producer seeks to maximize the present value of his firm (Devarajan and Go, 1996), which corresponds to the actualized value of his expected future profits (capital income) minus his expected future investment costs. This inter temporal trade-off decision can be formalised as follows:

$$\max \sum_{t} \frac{1}{(1+r)^{t-0}} \left( \hat{w}_{it}K_{it} -(1+X_{it})(\hat{P}_{it}I_{it}) \right)$$

$$\text{s.t. } K_{it+1} - K_{it} = -\delta_{it}K_{it} + I_{it}$$

Here $\hat{w}_{it}$ and $\hat{P}_{it}$ denote the expected unitary capital income and investment price. As we consider for now that the expectations are rational in the Muth sense, that is that they correspond to the “real” future values as defined by the economic model, we actually have: $\hat{w}_{it} = w_{it}$ and $\hat{P}_{it} = P_{it}$.

$X_{it}$ corresponds to the adjustment cost of capital: the cost of installing a new unit of capital and is equal to $\frac{\varphi}{2} \frac{P_{it}I_{it}^{2}}{K_{it}}$, with $\varphi$ an adjustment parameter (McKibbin and Wilcoxen, 1998).

The inter temporal optimisation program of the producer is thus:
\[
\max \sum_{t} \frac{1}{(1+r)^t} \left( wk_{ir t} K_{ir t} - \left(1 + \frac{\varphi}{2} \frac{PI_{ir t}}{K_{ir t}} \right) I_{ir t} \right)
\]
\[
st \ K_{ir t+1} - K_{ir t} = -\delta_\varphi K_{ir t} + I_{ir t}
\]

It corresponds to a Bolzano problem (the \( K_{ir t} \) variables are the state variables and the \( I_{ir t} \) variables are the control variables) and can be solved using the Pontryagin method:

Let \( H_{ir t} \) be the Hamiltonian of this problem:

\[
H_{ir t} = \sum_{t} \frac{1}{(1+r)^t} \left( wk_{ir t} K_{ir t} - \frac{\varphi}{2} \frac{PI_{ir t}}{K_{ir t}} \right) I_{ir t} + \pi_{ir t} \left(-\delta_\varphi K_{ir t} + I_{ir t}\right)
\]

With \( \pi_{ir t} \) the co-state variable corresponding to the implicit price of capital.

The corresponding first order conditions are:

\[
\frac{\partial H_{ir t}}{\partial I_{ir t}} = 0, \forall \ t \iff \frac{1}{(1+r)^t} \left(-\varphi PI_{ir t} \frac{I_{ir t}}{K_{ir t}} - PI_{ir t} \right) + \pi_{ir t} = 0 \iff \pi_{ir t} = \frac{PI_{ir t}}{(1+r)^t} \left(\varphi \frac{I_{ir t}}{K_{ir t}} + 1\right), \forall \ t
\]

\[
\frac{\partial H_{ir t}}{\partial K_{ir t}} = (\pi_{ir t} - \pi_{ir t-1}), \forall \ t \iff \frac{wk_{ir t}}{(1+r)^t} - \frac{\varphi PI_{ir t}}{2(1+r)^t} + \delta_\varphi \pi_{ir t} = \pi_{ir t} - \pi_{ir t-1}
\]

This leads to the following equation:

\[
k_{ir t+1} = \frac{\varphi}{2} \frac{PI_{ir t+1}}{K_{ir t+1}^2} + (\delta_\varphi - 1) PI_{ir t+1} \left(\varphi \frac{I_{ir t+1}}{K_{ir t+1}} + 1\right) + (1+r) PI_{ir t} \left(\varphi \frac{I_{ir t}}{K_{ir t}} + 1\right)
\]

If there were no adjustment costs (\( \varphi = 0 \)), this equation would be:

\[
k_{ir t+1} = (\delta_\varphi - 1) PI_{ir t+1} + (1+r) PI_{ir t}
\]

Wee here that the evolution of the price of investment depends on the future capital income. Indeed,

\[
\frac{PI_{ir t+1}}{PI_{ir t}} = \frac{wk_{ir t+1}}{(\delta_\varphi - 1) PI_{ir t}} - \left(\frac{1+r}{\delta_\varphi - 1}\right), \text{ so } \frac{PI_{ir t+1}}{PI_{ir t}} > 1 \iff \frac{wk_{ir t+1}}{(1+r)} > PI_{ir t} : \text{ if the actualised value of the future capital income is higher than the current price of investment, then the price of investment increases.}
\]

This equation will allow determining the level of investment which is now endogenous, contrary to the static model where it was set exogenously.
Once investment or capital stocks are known, the other producer’s decisions concerning output quantities and factor uses are taken as in the standard static model.

**Household’s decisions**

As producers make their investment decisions, households base their saving decision on an intertemporal trade-off. Indeed they spend a part of the income they earn at one period to consume goods, which brings them some utility, and save the remaining part of the income, which don’t bring them any utility for the ongoing period. However the part of the income saved at one period will be used later to consume and thus represents a future utility.

So, the representative household seeks to maximize value of his intertemporal utility. Here we assume that the utility function is additively separable, so the intertemporal utility function is equal to:

\[ U_n = \sum_t \frac{1}{(1+\rho)^t} u(Q_n), \text{ where } \rho \text{ is a time preference parameter (households have a preference for immediate utility).} \]

Furthermore, the household faces an intertemporal budget constraint which stands that the actualised value of all his future consumptions and savings cannot exceed the actualised value of all his incomes:

\[ \sum_t \frac{E_n}{(1+r)^t} \geq \sum_t \frac{1}{(1+r)^t} \left( \hat{P}_n Q_n + S_n \right). \hat{P}_n \text{ denotes the expected future price of consumption. As the expectations of households are supposed to be rational in this section, we have } \hat{P}_n = P_n. \]

So, the intertemporal optimisation program of the household is:

\[
\begin{align*}
\max & \quad U_n = \sum_t \frac{1}{(1+\rho)^t} u(Q_n) \\
\text{st} & \quad \sum_t \frac{E_n}{(1+r)^t} \geq \sum_t \frac{1}{(1+r)^t} (P_n Q_n + S_n)
\end{align*}
\]

The corresponding Lagrangian:
\[ L = \sum_{t} \frac{1}{(1+\rho)^t} u(Q_n) + \lambda \left( \sum_{t} \frac{1}{(1+r)^t} (E_{n} - P_{rt}Q_{nt} - S_{nt}) \right) \]

And the first order conditions:

\[
\frac{\partial L}{\partial Q_{nt}} = \frac{1}{(1+\rho)^t} u'(Q_n) - \lambda \frac{1}{(1+r)^t} P_{rt} = 0 \Rightarrow \lambda = \left( \frac{1+r}{(1+\rho)^t} \right) u'(Q_{nt}) \quad \forall t
\]

\[
\Rightarrow \frac{(1+r)^t}{(1+\rho)^t} P_{rt} = \frac{(1+r)^{t+1}}{(1+\rho)^{t+1}} u'(Q_{n+1})
\]

\[
\Leftrightarrow \frac{Q_{n+1}P_{n+1}}{Q_nP_n} = \frac{(1+r)}{(1+\rho)}
\]

So, solving the household’s program allows deriving an expression for the evolution of savings with time:

\[
\frac{E_{n+1} - S_{n+1}}{E_n - S_n} = \frac{(1+r)}{(1+\rho)}
\]

In fact this equation determines the evolution of savings from one period to another but not a level of savings at a given period independently from other periods. Actually the steady state assumption is very important here. Indeed at the steady state, savings equal investment: \( S_{rt} = PI_{rt}I_{rt} \) and this equality combined with the aforementioned equation allows deriving the level of savings for all the previous periods.

### 2.3 Summary of the main changes to the static model

Moving from the standard static GTAP AGR model to a dynamic model implies modifying the equations of the static model by notably adding a time dimension to the variables. In addition to these modifications to the existing equations, some new ones are added:

\[
w_{k_{ir+1}} = \frac{\phi}{2} P_{ir+1} \left( \frac{I_{ir+1}}{K_{ir+1}} \right)^2 + (\delta_r - 1) P_{ir+1} \left( \frac{\phi I_{ir+1}}{K_{ir+1}} + 1 \right) + (1+r) P_{ir} \left( \phi \frac{I_{ir}}{K_{ir}} + 1 \right) \quad t = 1 \text{ to } T - 1
\]

\[
w_{k_{ir}} = PI_{ir} (\delta_r + r) + PI_{ir} \phi \delta_r \left( \frac{\delta_r}{2} + r \right)
\]
3. Dynamic CGE model with non rational expectations

3.1 Non rational expectation schemes

Different points of view can be found on the way expectations are formed. According to Chavas (1999) expectation schemes can be classified in three groups: rational, quasi rational and naïve.

Many studies dealing with uncertainty assume rational expectations (Wright, 2001, Williams et Wright, 1991, Pratt et Blake, 2007), which means, as we already saw, that expected prices are those corresponding to the economic model (Muth, 1961). It is thus assumed that economic agents have the same knowledge than economists about markets functioning. Nerlove and Bessler (2001) argue that this assumption is in fact made most of the time because there no other theoretically acceptable assumption when dealing with aggregate behaviours. This, in addition to their good tractability, might explain why, to our knowledge, rational expectations are assumed in all the dynamic CGE models taking inter temporal behaviours into account.

However, according to some authors, expectations of economic agents are not rational in the Muth sense (Rosser and Kramer, 2001, Voituriez, 2001) due to information acquisition cost for instance. Non rational expectations, based on past information, seem in fact to better fit the way agricultural producers behave. In his 1999 study, Chavas concludes that this kind of expectations is the most
frequently among economic agents because of their capacity to collect and process information. Nerlove (1958) proposed a formalisation for expectations based on past information and adaptive. These Nerlovian expectations are such as agents take their past errors into account to form their new expectations: 

$$\hat{p}_t = \hat{p}_{t-1} + \alpha [p_{t-1} - \hat{p}_{t-1}] = \alpha p_{t-1} + (1-\alpha) \hat{p}_{t-1}, \quad 0 < \alpha \leq 1$$

can be seen as the weight of the previous period market price compared to all the earlier ones. In fact the lowest $\alpha$ is the greatest quantity of past information is taken into account.

An extreme case of Nerlovian expectation, opposite to the rational scheme, arises when $\alpha$ equals 1: the economic agent only considers the previous period to form his expectation. These are called naïve expectations. This assumption is often set to illustrate the Cobweb theorem which allows explaining the agricultural price volatility due to expectation errors from farmers (Butault and Le Mouël, 2004). Depending on the form of the supply and demand functions and on their parameters, the phenomenon described by the Cobweb theorem can lead to a convergence of market prices toward the equilibrium price or to perpetual cyclical fluctuations of the market (Mahé, 1977). It can even sometimes lead to chaos (Boussard, 1996).

The eventual chaotic behaviour of markets arising from the Nerlovian expectation scheme has raised criticisms against the non rational expectation assumption. Our objective in the remaining part of this paper is to build a dynamic CGE model considering this time Nerlovian instead of rational expectations, in order to see the impact of the expectation scheme on the results and to determine which factors can induce non convergent fluctuations of prices.

### 3.2 Construction of the model

**Iterative execution**

As we already mentioned, in the dynamic CGE model with rational expectations expected value are equal to the “true” future values. It implies that at the first period consumers and producers base their decisions on the “true” future market prices. Decisions taken during the second period rely on the
same future market prices and as a result lead to the same outcome as in the first period. Thereby there is no need to re-evaluate the model for each period: the solution for the first period corresponds to the optimal choice for the following periods (Ginsburgh and Keyzer, 1997). This need to solve the model for all the periods simultaneously can besides lead to some computational issues (Dixon et al., 2005).

On the other hand in the non rational expectation case, producers and consumers base their decision on expected prices which are not necessarily the true future market. Thus if during the second period they realize that their first period expectations were wrong, they will modify their expectations concerning the future periods. Thereby under the assumption of non rational expectations the model has to be re-evaluate for each period and only the results for the on going period matter because they will be used to form the next period expectations. The model is thus this iteratively solved, period by period.

As the capital stock of one period is a function of capital stock and investment of the previous period it therefore now exogenous: \[ \bar{K}_{t+1} = (1 - \delta) K_{t-1} + I_{t-1} \]. This fixity of the capital stock determines the unitary capital income.

**Production decision**

Contrary to consumers who face market prices when then they decide how much to consume, producers have to make expectations concerning the market price of their output when they decide how much to produce. That is why in our model Nerlovian expectation only concern producers. This was also pointed out by Turnovsky (1974) for instance.

The producer takes his production decisions according to his expectations concerning the factors and output prices. These expectations are Nerlovian:

\[
\hat{w}_{l_{t+1}} = \alpha \hat{w}_{l_{t+1}} + (1 - \alpha) \hat{w}_{l_{t+1}} \\
\hat{P}_{t+1} = \alpha \hat{P}_{t+1} + (1 - \alpha) \hat{P}_{t+1}
\]

He thus considers that the zero profit condition: \[ \hat{P}_{t+1} Y_{t+1} = \hat{w}_{l_{t+1}} L_{t+1} + \hat{k}_{w_{t+1}} K_{t+1} \], which together with the fixity of the capital stock allows him to infer a value for the capital remuneration \( \hat{w}_{k_{t+1}} \) and take his production decisions on that basis.
**Investment decision**

Even if their expectations are only based on past values, to take their investment decision producers have to anticipate investment costs and capital income for several future periods. Indeed let’s recall that the equations determining investment are

\[
\begin{align*}
\dot{w}k_{t+1} &= -\Phi \hat{p}I_{t+1} \frac{L_{t+1}}{K_{t+1}}^2 + (\delta_{t} - 1) \hat{p}I_{t+1} \left( \phi \frac{L_{t+1}}{K_{t+1}} + 1 \right) + (1+r) \hat{p}I_{t+1} \left( \phi \frac{L_{t}}{K_{t}} + 1 \right) t = 1 \text{ to } T - 1 \\
\dot{w}k_{t+1} &= \hat{p}I_{t+1} (\delta_{t} + r) + \hat{p}I_{t+1} \phi \delta_{t} \left( \frac{\delta_{t}}{2} + r \right)
\end{align*}
\]

Under the Nerlovian expectation assumption, the producer bases his expectation concerning one period on his observation of the previous periods. Yet in the case of investment decision this would imply him to observe the future investment costs, which is actually incompatible with the Nerlovian expectation scheme. To tackle this issue we assume that agricultural producers are myopic: they have expectations concerning the next one period and assume that these expectations prevail for all the future periods.

To illustrate that point consider the expectation of investment cost for the second period after the on going one: \( \hat{p}I_{t+2} = \alpha \hat{p}I_{t+1} + (1-\alpha) \hat{p}I_{t+1} \). The producer doesn’t observe \( I_{t+1} \), he thus has to make an expectation about it: \( \hat{p}I_{t+2} = \alpha \hat{p}I_{t+1} + (1-\alpha) \hat{p}I_{t+1} = \hat{p}I_{t+1} \). So in fact the producer’s expectations remain the same for all the future prices.

**Household behaviour**

Contrary to the producer who has to make expectations about market prices to take his production decision, the consumer faces the “true” market prices at the time to take his consumption decision. The equations determining consumption in the model thus remain unchanged.

Concerning savings, we consider for now that savings equalize to investment.
3.3 Summary of the main differences compared to the rational expectations model

The program is now solved iteratively, period by period. Within each period the model is divided (and executed) in two parts. In a first step the agricultural supply is determined on the basis of the agricultural producer’s expectations concerning the factors and output prices. The second step the solved the market equilibrium taking the agricultural supply has given.

The main changes stand in the first step of execution which allows determining the agricultural supply:

\[ \hat{P}_{ir} Y_{ir} = \hat{\omega} l_{ir} L_{ir} + \hat{\omega} k_{ir} \bar{K}_{ir} \quad \perp Y_{ir} \]

\[ \bar{K}_{ir} = K_{ir} \left( Y_{ir}, \hat{\omega} l_{ir}, \hat{\omega} k_{ir} \right) \quad \perp \hat{\omega} k_{ir} \quad (15) \]

\[ L_{ir} = L_{ir} \left( Y_{ir}, \hat{\omega} l_{ir}, \hat{\omega} k_{ir} \right) \quad \perp L_{ir} \quad (16) \]

With

\[ \bar{K}_{ir} = (1 - \delta) K_{ir-1} + L_{ir-1} \]

\[ \hat{P}_{ir} = \alpha P_{ir-1} + (1 - \alpha) \hat{P}_{ir-1} \]

\[ \hat{\omega} l_{ir} = \alpha \omega l_{ir-1} + (1 - \alpha) \hat{\omega} l_{ir-1} \]
4 Simulations

This last part is devoted to the results of some simulations we have conducted to compare the three models presented in the paper. These simulations are implemented in GAMS.

4.1 Calibrations

In the standard static version of the GTAP AGR model, the investment and the balance of trade are exogenous variables. Thereby this model doesn’t include the parameters required to represent the saving and investment behaviours we have in the dynamic version, namely the interest rate \( r \), the capital depreciation rate \( \delta \), the capital adjustment parameter \( \phi \), and the time preference parameter \( \rho \).

We thus have to calibrate these parameters. Furthermore savings and foreign debts are only implicitly taken into account in the standard model, so they are not included in the GTAP AGR database: we have to compute them.

For that purpose we start by setting \( r \) and \( \rho \) to 5% which is a value comprised in the interval estimated by Evans and Sezer (2005) for the EU countries.

The installation cost of one capital unit is also set to 5% of the value of this unit: \( X_{r0} = 0.05 \).

The values of investment in the GTAP database are aggregated at the regional level but in our model the capital accumulation and thus the investment are made at the sectoral level. However the database includes sectoral capital stocks and this provides a key to allocate regional investment by sector:

\[
P_{I_w} I_{w} = P_{I_r} I_{r} \times \frac{w_k}{w_k} K_{r}
\]

Then, because the static model can be viewed as a representation of markets at their steady state level, we consider that the initial GTAP database corresponds to a steady state. We thus know (equation 11)
that \( \frac{w_{kr}}{P_{ Ir}} = \delta_{ir} + r + \phi_{ir} \delta_{ir} \left( \frac{\delta_{ir} + r}{2} \right) \). Yet \( X_{ir} = \frac{\phi_{Ir}}{K_{ir}} \Rightarrow \delta_{ir} = 2X_{ir} \), so \( \frac{w_{kr}}{P_{ Ir}} = \delta_{ir} + r + X_{ir} \left( \delta_{ir} + 2r \right) \)

\( \Leftrightarrow \delta_{ir} = \frac{r \left(2 + X_{ir}\right)}{\frac{w_{kr}K_{ir}}{P_{Ir}I_{ir}} - (1 + X_{ir})} \) which provides the sectoral value of the capital depreciation rate.

We can then calibrate the \( \phi \) parameters thanks to the equation \( \phi_{ir} = \frac{2X_{ir}}{\delta_{ir}} \)

Furthermore what we have in the GTAP database is the value of the capital stock \( (K_{initial_{ir}} = w_{kr}K_{ir}) \) and not the quantity of capital. That is why, as Diao et al. (1998) did in their model, we re adjust the initial unitary capital income so that the following steady state condition is satisfied:

\( \frac{w_{kr}}{P_{Ir}} = \delta_{ir} + r + \phi_{ir} \delta_{ir} \left( \frac{\delta_{ir} + r}{2} \right) \Leftrightarrow w_{kr} = \delta_{ir} + r + \phi_{ir} \delta_{ir} \left( \frac{\delta_{ir} + r}{2} \right) \) (Otherwise \( w_{kr} \) would initially be set to 1 like all the other initial prices). Thereby \( K_{ir} = \frac{K_{initial_{ir}}}{w_{kr}} = \frac{K_{initial_{ir}}}{\delta_{ir} + r + \phi_{ir} \delta_{ir} \left( \frac{\delta_{ir} + r}{2} \right)} \)

At steady state savings equal investment: \( P_{r}I_{r} = S_{r} \Leftrightarrow I_{r} = S_{r} \) (because initially \( P_{r} = 1 \)), which allows determining the regional values of savings.

And finally at steady state the trade deficit equalizes the interests received on the foreign debt:

\( rD_{r} = -B_{r} \Rightarrow D_{r} = -\frac{B_{r}}{r} \). This allows computing the foreign debts values.

4.2 Scenario

The aim of the simulations we conduct in this paper is to compare the results of the static, dynamic with rational expectations and then dynamic with non rational expectations CGE models. We first focus on the static/rational dynamic comparison, the effect of the anticipation scheme being studied in the last part.
Our purpose is not for now to tackle current political issues but to compare different models on a simple same basis. That is why we choose to simulate the effects of the suppression of the European export subsidies in the cereal sector. The level of these subsidies is already quite low in the European Union, so the potential impacts of there suppression are of a limited political interest. However this kind of shock is precisely small enough to run our GAMS programs in a relatively short time and large enough to get significant results.

4.3 Results

*The standard static GTAP AGR model*

We first simulate the effects of the suppression of European export subsidies in the cereal sectors with the standard static model. This suppression occurs in four sectors: rice, wheat, oilseeds and other cereals. However for a better readability of the presentation we will focus here on the results concerning the sectors where the level of production in European Union is initially the highest which are also the sectors where the largest part of production is exported, namely the wheat and other cereals sectors. These results are reported in table1.

**Table1: Impacts of the shock on the European cereal sectors - static model**

<table>
<thead>
<tr>
<th>% change</th>
<th>Export</th>
<th>Output</th>
<th>Price</th>
<th>Capital stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wheat</td>
<td>-14,7</td>
<td>-5,5</td>
<td>-0,8</td>
<td>-5,4</td>
</tr>
<tr>
<td>Other Cereals</td>
<td>-18,5</td>
<td>-5</td>
<td>-0,8</td>
<td>-4,9</td>
</tr>
</tbody>
</table>

We find here the classical results that a suppression of export subsidies leads to a decrease of exports in the concerned sectors. As exports decrease domestic production decreases too but in a lesser extent (the output decrease compensates 96% the export decrease in the wheat sector and 98% in the other cereals sector), this leads to a price decrease. Finally, the value of capital stocks depreciates.
The dynamic model with rational expectations

We now rely on the dynamic model with rational expectations.

The suppression of export subsidies occurs in the first period and we assume that the steady state is reached 15 periods after the shock; the model is thus run for 15 periods.

Table 2 and 3 report the results for the wheat and other cereals sectors.

Table 2: Impacts of the shock on the European wheat sector – dynamic model with rational expectations

<table>
<thead>
<tr>
<th>Period</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>13</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>Export quantity (%)</td>
<td>0</td>
<td>-13.2</td>
<td>-14</td>
<td>-14.4</td>
<td>-14.6</td>
<td>-14.7</td>
<td>-14.7</td>
<td>-14.7</td>
</tr>
<tr>
<td>Output quantity (%)</td>
<td>0</td>
<td>-4.7</td>
<td>-5.1</td>
<td>-5.3</td>
<td>-5.2</td>
<td>-5.5</td>
<td>-5.5</td>
<td>-5.5</td>
</tr>
<tr>
<td>Output price (%)</td>
<td>0</td>
<td>-1.3</td>
<td>-1.1</td>
<td>-0.9</td>
<td>-0.9</td>
<td>-0.8</td>
<td>-0.8</td>
<td>-0.8</td>
</tr>
<tr>
<td>Capital stock (%)</td>
<td>0</td>
<td>-2.3</td>
<td>-3.8</td>
<td>-4.6</td>
<td>-5</td>
<td>-5.2</td>
<td>-5.3</td>
<td>-5.4</td>
</tr>
</tbody>
</table>

Several observations come out from these results. First, for the wheat as well as for the other cereals sectors the percentage changes in export, production, price and capital stock value at the last period correspond to those found with the static model. This result is not surprising since this last period corresponds to a steady state and, as we previously advocated, the static model offers a representation of markets at their steady state level: the “after shock” steady states are the same for the static than for the dynamic model. Then, the evolution of market variables over time appears to be linear, there are no fluctuations. This comes from the fact that, because of the rational expectations assumption, the evolution of capital stock, on which the dynamics of the model are based, is optimized from the first period on the steady state path. Thus the model doesn’t allow representing the market fluctuations over
time, which is yet one of the aim of the use of dynamic modelling, when dealing the issues of risk and market stabilisation for instance.

So the use of a dynamic CGE model with rational expectations seems to be of a limited interest because its results don’t bring much more information than those of a static model for an execution time of the program much more important (some seconds for the static model and at least half an hour for the dynamic model). This lack of significant differences between the two approaches was also pointed out by Rutherford and Tarr (2003).

At last the results bring out the fact that static CGE models provide a picture of markets several years after the simulated shock. Here for instance the dynamic model reaches the values provided by the static model some 11 years after the shock. However, although they have been created for that purpose, static CGE models are often used to conduct short term analysis (Hertel et al., 2005). Thereby when using these models one must always have in mind that they are useful tools to value the long term effects of political reforms but should by no means be used to assess their short term impacts.

The dynamic model with non rational expectations

Having noticed that the dynamic modelling did not bring results very different from those of the static modelling when rational expectations were assumed, we now focus on the non rational case in order to see whether or not the anticipation scheme can have an impact on the results of the dynamic CGE model.

Let’s recall that agricultural producer’s expectations concerning the output price and the factor income are now Nerlovian: $\hat{P}_{it} = \alpha P_{it-1} + (1-\alpha) \hat{P}_{it-1}$ and $\hat{w}_{it} = \alpha \hat{w}_{it-1} + (1-\alpha) \hat{w}_{it-1}$, with $0 < \alpha \leq 1$

The closest to 1 is $\alpha$, the more naïve are the expectations.

A first observation is that for $\alpha > 1/5$ the model cannot be solved. To illustrate this phenomenon we have represented the evolution of the wheat price of (figure1 below). What appears is that the variations of output quantities during the first periods after the shock are increasing with $\alpha$. There
seems to be a convergence toward on output decrease of 0.8% for $\alpha \leq 1/5$, but the opposite phenomenon arises for $\alpha > 1/5$. When $\alpha = 1/5 + 1/10 = 21/100$ for instance fluctuations of price increase with time and become so high 6 periods after the shock that the model cannot be solved: the evolution of markets leads to chaos.

**Figure 1: Fluctuations of wheat price over time for different value of alpha (%change)**

![Fluctuations of wheat price over time for different value of alpha](image)

In fact Nerlove himself in his seminal work on adaptive expectations (Nerlove, 1958), provides an explanation to this chaotic behaviour of market: it notably depends on the supply elasticities. Indeed the more responsive is the agricultural production to price changes the higher are its fluctuations. Besides as the static GTAP AGR model deal with long term behaviour the elasticities parameter are quite high. However it is well know that in the short term the agricultural is quite inelastic because once the production decisions are taken it becomes difficult to move production factors and adjust the output quantities. To take this important point into account we divide by two the elasticities of substitution between primary factors and the elasticities of substitution between factors and intermediate consumption in the agricultural sector in the European Union: the elasticity of
substitution between primary factors is set to 0.2 instead of 0.4 in the static model, and the elasticity of substitution between factors and intermediate consumption is set to 0.4 instead of 0.9.

These new elasticities allow the model converging for higher values of $\alpha$, also the naïve case of $\alpha = 1$ cannot be reached. Indeed, contrary to what we obtained with the former elasticities, the model can now be solved for values of $\alpha$ comprised between 1/5 and 1/4. However higher values still conduct to chaos.

The results concerning changes in output price and quantity, obtained with various values of $\alpha$ are reported in tables 4 and 5 below.

**Table 4: Impacts of the shock on the European wheat sector – dynamic model with Nerlovian expectations**

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>13</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>9/40</td>
<td>-4.3</td>
<td>-1.3</td>
<td>0.1</td>
<td>-0.9</td>
<td>-1.9</td>
<td>-1.7</td>
<td>-0.9</td>
<td>-0.7</td>
</tr>
<tr>
<td>1/5</td>
<td>-4.3</td>
<td>-1.6</td>
<td>-0.2</td>
<td>-0.6</td>
<td>-1.1</td>
<td>-2.2</td>
<td>-1.0</td>
<td>-0.4</td>
</tr>
<tr>
<td>1/6</td>
<td>-1.4</td>
<td>-0.7</td>
<td>-1.3</td>
<td>-0.9</td>
<td>-1.3</td>
<td>-1.1</td>
<td>-1.2</td>
<td>-0.9</td>
</tr>
<tr>
<td>1/8</td>
<td>-1.4</td>
<td>-0.9</td>
<td>-1.3</td>
<td>-0.9</td>
<td>-1.3</td>
<td>-1.0</td>
<td>-1.2</td>
<td>-1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Output quantity (%)</th>
<th>9/40</th>
<th>1/5</th>
<th>1/6</th>
<th>1/8</th>
</tr>
</thead>
<tbody>
<tr>
<td>9/40</td>
<td>-1.4</td>
<td>-4.3</td>
<td>-6.4</td>
<td>-5.8</td>
</tr>
<tr>
<td>1/5</td>
<td>-1.2</td>
<td>-1.5</td>
<td>-0.8</td>
<td>-0.9</td>
</tr>
<tr>
<td>1/6</td>
<td>-1.3</td>
<td>-0.9</td>
<td>-1.2</td>
<td>-1.0</td>
</tr>
<tr>
<td>1/8</td>
<td>-5.0</td>
<td>-5.1</td>
<td>-5.0</td>
<td>-5.1</td>
</tr>
</tbody>
</table>

**Table 5: Impacts of the shock on the European “other cereals” sector – dynamic model with Nerlovian expectations**

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
<th>11</th>
<th>13</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>9/40</td>
<td>-8.1</td>
<td>3.2</td>
<td>-0.1</td>
<td>-5.0</td>
<td>1.7</td>
<td>-0.7</td>
<td>-3.2</td>
<td>0.4</td>
</tr>
<tr>
<td>1/5</td>
<td>-8.1</td>
<td>2.4</td>
<td>1.3</td>
<td>-5.4</td>
<td>0.3</td>
<td>1.6</td>
<td>-3.6</td>
<td>-1.3</td>
</tr>
<tr>
<td>1/6</td>
<td>-1.2</td>
<td>-1.5</td>
<td>-0.8</td>
<td>-0.9</td>
<td>-1.2</td>
<td>-1.3</td>
<td>-0.7</td>
<td>-1.2</td>
</tr>
<tr>
<td>1/8</td>
<td>-1.3</td>
<td>-1.2</td>
<td>-1.2</td>
<td>-1.0</td>
<td>-1.2</td>
<td>-1.0</td>
<td>-1.0</td>
<td>-1.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Output quantity (%)</th>
<th>9/40</th>
<th>1/5</th>
<th>1/6</th>
<th>1/8</th>
</tr>
</thead>
<tbody>
<tr>
<td>9/40</td>
<td>-4.7</td>
<td>-6.8</td>
<td>-6.3</td>
<td>-4.9</td>
</tr>
<tr>
<td>1/5</td>
<td>-4.7</td>
<td>-6.8</td>
<td>-6.3</td>
<td>-4.9</td>
</tr>
<tr>
<td>1/6</td>
<td>-4.7</td>
<td>-4.5</td>
<td>-4.9</td>
<td>-4.9</td>
</tr>
<tr>
<td>1/8</td>
<td>-4.7</td>
<td>-4.8</td>
<td>-4.8</td>
<td>-4.8</td>
</tr>
</tbody>
</table>

The results concerning the 15th period after the suppression of export subsidies were identical with the static model and with the dynamic model with rational expectations: a 5.5% decrease of production and a 0.8% decrease of price for wheat, and a 5% decrease of production and a 0.8% for other cereals.
Assuming non rational adaptive expectations lead to different results: 15 periods after the shock the production decrease ranges between 6% and 5% for wheat and between 5.6% and 4.6% for other cereals, and the impact on price ranges between a 1% decrease and a 0.4% decrease for wheat and between a 1.3% decrease and a 0.4% increase for other cereals, depending on the value of the $\alpha$ parameter. Just focusing on the results for this last period it doesn’t seem that any value of $\alpha$ leads to results closest to those obtained under rational expectations than other values of $\alpha$. On the other hand the computation of variances of output changes over the 15 periods allows highlighting the role of the parameter $\alpha$. Indeed for $\alpha = 9/40$ or $\alpha = 1/5$ variances of output changes are around 2.85 for wheat and 4.2 for other cereals, whereas for $\alpha = 1/6$ or $\alpha = 1/8$ these variances are almost null for both sectors. So, as $\alpha$ increases, that is as the weight given to past values diminishes in the formation of expectations, the suppression of export subsidies generates more fluctuation of output quantities. The computation of price variances leads to the same conclusion.

This is even more obvious on the graphical representation of the evolution of capital stocks (figure 2 below). In fact it appears that under the assumption of rational expectations the value of the capital stocks evolve continuously toward a decrease of 5.5% for the wheat sector and of 5% for the other cereals sector; whereas when Nerlovian expectations are assumed the evolution occurs through fluctuations and these fluctuations are all the more high as expectations tend to be naïve.

**Figure2: evolution of capital stocks for different anticipation schemes (%change)**
One can also notice that whatever the value of $\alpha$ is, the magnitude of fluctuations tends to decrease with time.
Conclusion

Most of the CGE models used today to value the effects of political reforms are static or are said to be dynamic but don’t take the inter temporal decision process of economic agents into account. In this paper we have departed from the well know static GTAP AGR model and built a dynamic model including inter temporal optimisation programs of producers and consumers to determine their investment/saving behaviours. Considering that agents integrate the future outcomes of their choices in their decision process requires modelling the way they form their expectations. The nature of expectation schemes has already been extensively discussed in the economic literature particularly during the 70’s and 80’s, but most of the time what is assumed in the economic modelling is that agents behave rationally. However agricultural producers have to anticipate the price at which they will sell their production to decide how much they will product. And, as shown by Ezekiel (1938) in his Cobweb theorem, if these producers are not rational but form their expectations on the base of past information, this can induce market fluctuations. This endogeneity of market risk is one the argument in favour of a public intervention to stabilize agricultural markets, therefore it is a crucial point. We have thus built two dynamic CGE models: one of them assumes rational expectations (as has been done by Diao and Somwaru, 2000 or Devarajan and Go, 1998) and the other one assumes adaptive Nerlovian expectations for agricultural producers. To compare these two models we have simulated a relatively small political shock: the suppression of export subsidies in the European cereal sectors. What comes out of these simulations is that the dynamic model with rational expectations leads the same results as the static model: markets evolve linearly toward a steady state which corresponds to the static situation after the shock. On the other hand under the non rational expectations assumption the political shock generates market fluctuations. These fluctuations are all the more high that expectations take account of few past information and can even become higher and higher with time which leads to chaos if expectations are too naïve.

This work represents a first step toward the integration of market risk in CGE modelling. And according to the current evolution of agricultural policies toward subsidies more and more decoupled from
production and prices, this acknowledgement of risk is essential for economic evaluations. The dynamic model with non rational expectations allows simulating exogenous (due to a political shock for instance) and endogenous (following the shock) market fluctuations. The next steps will be, first to identify exactly which expectation scheme better fits the behaviour of economic agents, second to introduce the reaction of at least agricultural producers to those market risks in the model by introducing risk aversion and relying on the expected utility theory for instance, and third, in the line of Hertel et al. (2005), to introduce stockholding behaviours in the model. Indeed stockholding behaviours consist in buying stocks when prices are low and selling them when prices increase, which mitigates these increases, and can thus lead to reduce the market fluctuations, to smooth the distributions of prices and so to moderate the high fluctuations sometimes induced by the Nerlovian expectations assumption. Finally, in the models presented here we have considered households and producers as two different economic agents; some improvement could certainly be done by taking into account the fact that agricultural households are producers and consumers at the same time. Indeed, these households have to take investment as well as savings decisions and this should affect the results of the dynamic models.


