Supplementary material 1. Using a proposal distribution based on the multiplication by a log-normally distributed variable.

We consider the relation between to random variables $X$ and $Y$, $Y = X \times LN(0,0.1)$. We need to compute the ratio

$$\frac{Pr(Y|X)}{Pr(X|Y)}$$

which is used in several acceptation ratios of the MCMC developed for this study (e.g. Step 1 in the Appendix “Bayesian estimate through a MCMC algorithm”).

Because the ratio $Y/X$ follows a log-normal distribution with parameters ($\mu=0$, $\sigma^2=0.1$) we have

$$Pr_Y(y|X)dy = Pr(Y = y \pm dy|X) = Pr\left(Z = \frac{y}{X} \pm \frac{dy}{X}|X\right) = Pr_{LN}\left(\frac{y}{X}|X\right)dy.$$  

Using the PDF of a log-normal, then

$$Pr(Y|X) = \frac{1}{\sqrt{2\pi} \sigma Y/X} \exp\left(-\frac{(\log Y/X - \mu)^2}{2\sigma^2}\right) \frac{1}{X},$$

and the ratio simplifies to

$$\frac{Pr(Y|X)}{Pr(X|Y)} = \frac{X}{Y}.$$ 

Now, this expression is used to compute the acceptation ratio, $r_\alpha$, defined in Step 1 of the Appendix “Bayesian estimate through a MCMC algorithm” that becomes

$$r_\alpha = \frac{Pr(F|\sigma_t)}{Pr(F|\sigma_t \times Prior(\sigma_t))} \frac{\sigma_t \times Prior(\sigma_t)}{Pr(F|\sigma_t \times Prior(\sigma_t))}.$$ 

Because we chose a prior distribution defined as $Prior(\sigma_t) \times \frac{1}{\sigma_t}$, the acceptation ratio $r_\alpha$ simplifies to

$$r_\alpha = \frac{Pr(F|\sigma_t)}{Pr(F|\sigma_t)}.$$